3601. The shortest path between a point and a curve lies along a normal to the curve. So, find the equation of a generic normal to $y = 1/x$ at $x = p$, and use this to find the equations of the normals.

3602. Draw a force diagram. Split isosceles triangle BAP up into two right-angled triangles, and let $2\theta = \angle BAP$. Find $\sin\theta$ in terms of l. Then take moments around A and use identities to simplify.

3603. For both parts, write the terms of the sequence longhand, and consider the possibility space as the ${}^{10}\mathrm{C}_2 = 45$ possible pairs chosen from the list.

3604. (a) Express x^2 as $A(3x+1)^2 + B(3x+1) + C$ and equate coefficients.
(b) Integrate term by term.

3605. Consider $y = \arccos x$ as the reflection of $y = \cos x$ in $y = x$. Find the appropriate tangent line on $y = \cos x$ and reflect it in $y = x$.

3606. Find the probability distribution of n, which can take values $1, 2, 3, 4$. Find the probabilities of $n = 1, 3, 4$ first, and hence $n = 2$. Then work out the expectation.

3607. Rotation clockwise around O by 90° is the same as

① reflection in $y = x$, followed by
② reflection in the x axis.

Perform these transformations algebraically.

3608. $a = 2$ and $b = -1$ is a counterexample to two of the implications.

3609. Set up the integral

$$A = 4\int_0^r \sqrt{r^2 - x^2}\,dx.$$

Use the substitution $x = r\sin\theta$.

3610. This is a cubic in $\sqrt{x}$. Let $z = \sqrt{x}$.

3611. Let $z = x^2 - y^2$. Substitute for x, and then set $\frac{dz}{dy} = 0$ to maximise z.

3612. Integrate separately term by term. The constant 1 can be integrated in standard fashion, and $\mathrm{g}(x)$ can be integrated using the given symmetry. A sketch will help.

3613. (a) Differentiate the rate with respect to t, and set the second derivative to zero.
(b) Consider the integral as a continuous sum.
(c) Integrate by parts with $u = t$ and $\frac{dv}{dt} = e^{p - \frac{1}{q}t}$.

3614. (a) Use the length of the string to find the angle between the two radii. This will allow you to include the angle θ on both diagrams.
(b) Resolve along the string.

3615. Take the LHS and differentiate by the chain rule. Then take the RHS and simplify using a double-angle formula (choose the one that will get rid of the -1). Show that the two sides are equivalent.

3616. (a) The successful outcomes begin with $N = 9, 10$.
(b) Consider the successful outcomes from (a) as the possibility space.

3617. Factorise fully, and consider the multiplicity of the roots. Also consider the degree of the curve, and whether it is positive or negative.

3618. Show that an equilateral triangle of side length 33 cm won't fit.

3619. (a) Carry out the two calculations.
(b) The curve $y = x^3 - x$ has rotational symmetry around the origin.

3620. Let $z = e^x$ and $y = \ln x$. Calculate derivatives with respect to x. Then find $\frac{dz}{dy}$ using the chain rule.

3621. (a) There should be four forces, with magnitudes R_1, μR_1, R_2 and mg.
(b) Assume wlog the ladder has length 2 m.
(c) Work out the reaction at the wall, using the moments equation. This will give you two equations. Eliminate the reaction at the floor to get μ.

3622. Sketch the loci carefully, noting the symmetry in $y = x$. Find the shortest distances by considering normals.

3623. (a) Express the integrand of I in partial fractions. Then simplify with a log rule, such that you can simplify e^I.
(b) Consider that $A = e^c$.

3624. It is possible for the tests to yield different results. Consider the relative sizes of the critical regions.

3625. Set up a formula for A in terms of θ. Differentiate it (implicitly) with respect to t. Then substitute in $\frac{d\theta}{dt} = 1$ and use a small-angle approximation.

3626. Differentiate with respect to y. Use this to find the equation of the tangent at $y = 4$, again with x as the subject. Substitute for x and then solve for intersections.

3627. The generating of roots beyond the first has been left too late. Four possibilities should appear in the second line.

3628. Show that the denominator has no roots. Then find the coordinates of the two SPs and the two axis intercepts. Consider the behaviour as $x \to \pm\infty$, and put the picture together.

3629. Consider the trilogy as a single object. Find the number of ways of rearranging the eight objects in all, and then multiply by the number of ways of rearranging the trilogy amongst itself.

3630. Set up a the required equation, and multiply up by the denominators. Then consider the prime factors of the LHS and RHS.

3631. (a) Use $\dfrac{1}{r(r+1)} \equiv \dfrac{A}{r} + \dfrac{B}{r+1}$.
(b) Write the sum out longhand.
(c) Combine the non-cancelled terms.

3632. (a) Factorise and consider the multiplicity of the roots at $x = 0$ and $x = 1$.
(b) Set up and carry out a definite integral.

3633. Rearrange to make y the subject. Find the x axis intercepts, then set up a definite integral.

3634. Use a calculator polynomial solver to find roots, and then the factor theorem to convert those roots into factors.

3635. Multiply the equation out, and consider it as a quadratic in y.

3636. Find the sums of

- the first 100 integers,
- the first 25 multiples of 4,
- the first 20 multiples of 5,
- the first 5 multiples of 20.

3637. (a) Consider the forces exerted on the two pulleys by the string.
(b) Using your answer to (a), set up an equation of motion along the string.

3638. Express the curve as a transformation of $y = \frac{1}{x}$. You can quote the fact that this has no points of inflection.

3639. (a) Find the probability that (X_1, X_2) is $(0, 3)$ or $(1, 2)$ or vice versa.
(b) Consider the fact that the variable $X_1 + X_2$ is itself distributed binomially.

3640. Let $u = x^3$ and $v' = x^2\sqrt{x^3+1}$. Integrate the latter by inspection (reverse chain rule) to find v. Then substitute into the parts formula. You'll need to integrate by inspection again.

3641. Write the LHS in harmonic form.

3642. Note that the differences between the times are $36 - 12 = 24$ and $60 - 36 = 24$. This allows you to calculate a directly, without setting up a specific algebraic model.

3643. Use the sum-product identities on the RHS, and simplify to reach the LHS.

3644. (a) Look for the maximum value for $t \in [0, 4]$ and then also for $t \in [4, \infty)$.
(b) Integrate the velocity piecewise, i.e. over $[0, 4]$ and then $[4, 10]$.

3645. This is a quadratic in $\ln x$. Writing $\ln 128 = 7\ln 2$, it can be factorised.

3646. You need to find the volume of the region on the far side of ABC. Set the origin as the far vertex of the base and give the points A, B, C coordinates $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$. The average of these gives the centre of $\triangle ABC$. You can then find its height (distance to the origin) using Pythagoras. Then use a binomial probability.

3647. (a) Consider the magnitude of the integrand as $x \to \infty$.
(b) Carry out the trapezium rule and compare the result to $\pi^2/6$.

3648. Determine the equation of the chord. Then solve it simultaneously with the equation of the curve, and look for double roots.

3649. Consider a unit square as the possibility space, with area representing probability.

3650. Find the y coordinate at $x = a$. Then find the equation of the line through the origin and this point. Solve it simultaneously with the curve, looking for double roots.

— Alternative Method —

Differentiate by the quotient rule. Then find the gradient and the y coordinate at $x = a$. Find the generic equation of the tangent at $x = a$.

3651. Undo the sine function and find the equations of all the lines whose points satisfy $\sin y = \sin 2x$. Sketch these, and determine the coordinates of one of the quadrilaterals so produced.

3652. This is false. For example, consider $y = x$, $y = 2x$ and a third equation, for $n = 2$.

3653. (a) Differentiate implicitly, using the chain rule.
(b) Substitute the relevant values in.

3654. Write the cube root as

$$x\left(1 + \frac{h}{x^3}\right)^{\frac{1}{3}}.$$

Expand this binomially. The terms independent of h then cancel, allowing you to divide top and bottom by h and so take the limit.

3655. Find the probability that A_1 and A_2 sit together, and that no other couple does. Multiply by 3.

3656. (a) Count up the total number of vertices, and then divide by the overcounting factor 3 (three faces meeting at every vertex.)
(b) Do similarly.
(c) Substitute into the formula $V - E + F = 2$.

3657. Use the sketch, and the value $\text{g}(0)$, to find the range of g. Hence, find the domain and codomain of the invertible version of g. Switch these for the domain and codomain of g^{-1}. Find the algebraic instruction by setting up $y = \text{g}(x)$ and solving for x. Choose the positive square root when the time comes.

3658. Turn the symmetry statement into an algebraic equation. Then integrate it.

3659. Take, as a possibility space, the smallest square which contains one of the octagons.

3660. Square the equations and add them, simplifying by the first Pythagorean trig identity. Then multiply out and simplify.

3661. Neither asymptote is vertical. They occur as $x \to \pm\infty$.

3662. Having differentiated with respect to y, use the second Pythagorean trig identity.

3663. Taking out a factor of $x^{0.1}$ leaves a quadratic in $x^{0.3}$.

3664. Show that $y = \text{h}(x)$ can have no more than one turning point.

3665. (a) Consider the values of the common ratio for which a geometric sum to infinity converges.
(b) Consider the range of S_∞.

3666. Sketch the boundary equations first, which are two intersecting ellipses. Then consider the signs of the factors: you need exactly one to be positive and one negative.

3667. (a) Draw a clear sketch.
(b) Take out the factor of $\frac{1}{n^3}$ from each sum, then use the given result.
(c) Show that both bounds on A tend to the same result.

3668. Consider a possibility space of 5^4 outcomes. Out of these, successful regions contains four of the colours, in some order.

— Alternative Method —

For a conditioning method, colour one region wlog. Colour another region, finding the probability of success. Continue likewise through the regions, multiplying probabilities.

3669. For both (a) and (b), note that, for the purposes of NII, moving at constant speed in a straight line is equivalent to equilibrium.
(a) Write down the result.
(b) Draw a force diagram.

3670. Write the integrand as a proper algebraic fraction, then integrate. Remember the modulus function inside the natural logarithm, you'll need it here.

3671. This is a quadratic in x^2y. Put everything onto the LHS, and factorise. Then sketch two graphs of the form $y = k/x^2$.

3672. Find the equation of a generic tangent to the curve at $x = p$. Then substitute $(57/8, 0)$ into this and solve for p.

3673. (a) Consider a transformation of $y = \tan x$, which has period π.
(b) Work out the period of $\operatorname{cosec} x$, then $\operatorname{cosec} 2x$. Then combine the two periods using a lowest common multiple.

3674. Put everything onto the LHS and factorise, using the fact that $e^{x+y} \equiv e^x e^y$.

3675. Draw a force diagram, labelling the magnitudes of the reaction forces R_1 and R_2. Set up equations for vertical equilibrium and moments around e.g. the right-hand end. Solve these for $R_1 + R_2$. Note that you don't have to find the individual reaction forces.

3676. (a) Equate the denominator to zero.
(b) You don't need to differentiate here, although you could. Consider the range of numerator and denominator.
(c) Substitute the x values of the stationary points into the second derivative, and interpret the results.
(d) Put it all together!

3677. Choose one face, without loss of generality. Then place the other two, multiplying the probabilities of success.

——— Alternative Method ———

Successful outcomes can be counted as the number of vertices of the dodecahedron.

3678. Consider separately limits $x \to 2^+$ and $x \to 2^-$, in which the notation means "x tends to 2 from above" and "x tends to 2 from below".

With a direction chosen, you can get rid of the mod function, and evaluate a one-sided limit.

3679. (a) Solve the boundary equation.
(b) With the RHS factorised, square both sides, noting that, wherever the inequality in (a) is satisfied, you introduce new solution points by doing so.

3680. Consider the constant of integration.

3681. Use a small-angle approximation. Then expand $(1+2x)^7$ and $(1+3x)^{-4}$ binomially, ignoring terms in x^3 and higher. Then multiply these out, again ignoring terms in x^3 and higher.

3682. Only one of these is true.

3683. Set up the equation for intersections: you should find a quadratic in x. Find the discriminant in terms of p and q. Hence, write down values of p and q for which the discriminant is negative.

3684. Differentiate to find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. Set each of these to zero and solve. Show that there are t values in common to both solution sets.

3685. (a) The total frequency, i.e. the total area, must be 1 million.
(b) The frequency density is greatest at the max of $\text{h}(x)$.
(c) Set up a definite integral, with limits 0 and m, and equate it to $\frac{1}{2}$. Carry out the integral to reach a quadratic in m^4. Solve this, choosing the appropriate root.

3686. Call the side lengths $(a, a, 2b)$. Split the triangle into two right-angled triangles, and set up a pair of simultaneous equations.

3687. (a) Solve the boundary equations simultaneously, considering the latter equation as $x - y = \pm 1$.
(b) The curves are a hyperbola and a pair of straight lines, and the region R contains the origin.
(c) Integrate $^2/_x$ from $x = 1$ to $x = 2$ to find an area bounded by the curved edge of R. Then add and subtract various triangles and squares, and use symmetry.

3688. Differentiate using the chain rule, and simplify as far as you can. Then explain why, if $n \in (0, 1)$, the values $x = 0$ and $x = 1$ both leave the gradient undefined. Note that, in the algebra, the ways in which $x = 0$ and $x = 1$ render the gradient infinite are different.

3689. Drop a perpendicular from E to AB. The length of this is $\sin 2\theta$. Establish its length by finding $|BC|$ and $|CD|$.

3690. (a) Consider the domain of $\ln x$.
(b) Differentiate by the product rule.
(c) The notation $x \to 0^+$ means that x is heading for 0 from above. From (b), you know that, as x does this, the value of $\text{f}(x)$ is increasing. Consider this graphically.

3691. Find the acceleration, i.e. the second derivative of x with respect to t. Then substitute into $6x + a$, and take out a factor of $(6 - \omega^2)$.

3692. Show that the curve has no stationary points. Hence, take the domain at its largest possible, namely $(1, \infty)$. Find the range by considering $x \to 1^+$ and $x \to \infty$. Set the codomain of f to this range.

3693. Draw an unwrapped picture of the cylinder, as if you've cut it lengthwise and laid it flat on the table to form a rectangle. Once you've got the picture, it's a single trigonometric calculation to find the relevant angle.

3694. (a) Set x or y to zero.
(b) Set the first derivative (product rule) to zero, and classify using the second derivative.
(c) Note that an exponential term dominates any polynomial term.
(d) Join the dots!

3695. Give a counterexample, i.e. draw a map which clearly requires four colours to shade it.

3696. A curve has a horizontal asymptote if, when x tends to either plus or minus infinity, y tends to a constant. Two are true and one is false.

3697. Use the identity $\cos 2x \equiv 1 - 2\sin^2 x$.

3698. 4 can be obtained by rolling

① four, then four sixes from four rolls,
② five, then four sixes from five rolls,
③ six, then three sixes from six rolls.

3699. (a) Substitute in and use the first Pythagorean trig identity.

(b) Using the parametric differentiation formula and the chain rule.

(c) Find the relevant value of t, and substitute into $\frac{dy}{dx}$. Then use $y - y_1 = m(x - x_1)$.

3700. Rarely, a force diagram won't help much, since none of the forces are given explicitly. Instead, proceed algebraically. Add the three forces and solve for a and b. Then find the equations of the lines of action of the first two forces. Solve these simultaneously. Then engineer the location of the third force so that its line of action is concurrent with the other two.

——— End of 37th Hundred ———